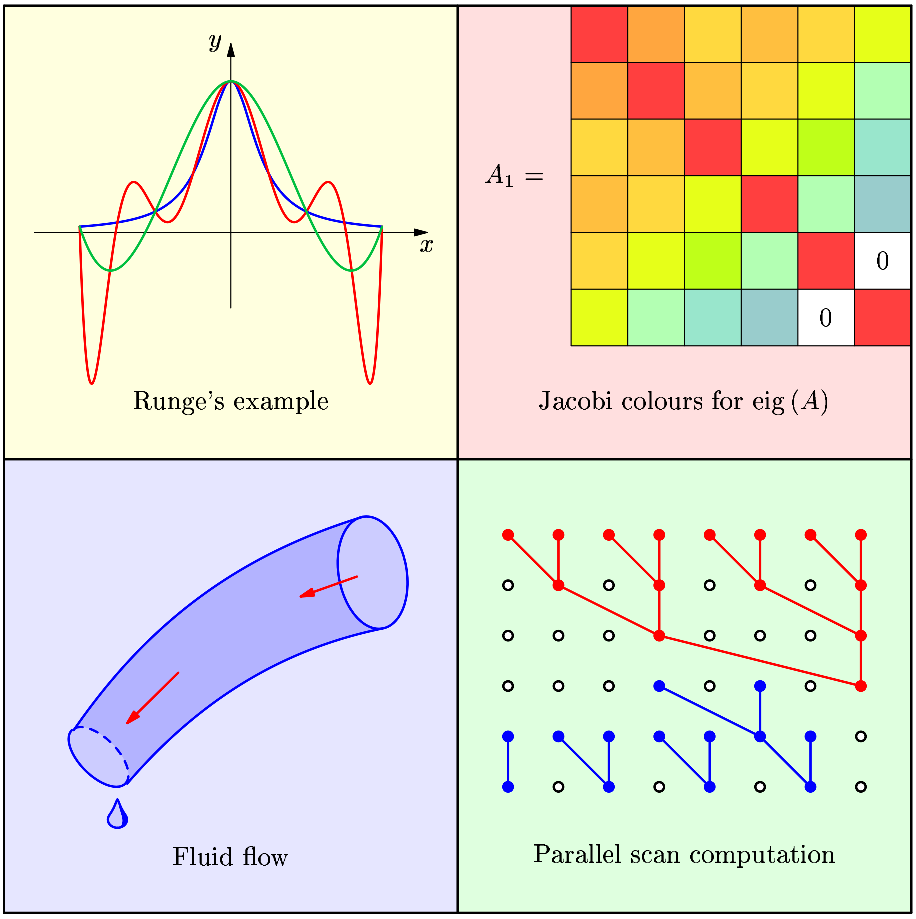
APPLIED MATHEMATICS AND SCIENTIFIC COMPUTING

Brijuni, Croatia
June 19–24, 2005.



Runge's example

Jacobi colours for eig (A)

Fluid flow

Parallel scan computation

SCIENTIFIC PROGRAM

Organized by:

Department of Mathematics, Unversity of Zagreb, Croatia.

Eduard Marušić–Paloka, chairman

Organizing committee:

Mladen Jurak
Josip Tambača
Miljenko Marušić
Mladen Rogina
Zlatko Drmač
Sanja Marušić
Saša Singer

Scientific committee:

Alain Bourgeat
Sunčica Čanić
Zvonimir Tutek
Luka Sopta
Rudolf Scitovski
Krešimir Veselić
Andro Mikelić

Sponsors:

Ministry of Science and Technology, Croatia,
Element d.o.o. za nakladništvo
CV Sistemi d.o.o., Padovčeva 3, 10000 Zagreb, Croatia.

Monday, June 20

Time	Section A
9:00– 9:15	**Conference opening**
9:15–10:00	Sunčica Čanić*, Giovanna Guidoboni, Andro Mikelić and Josip Tambača An Effective Model of Blood Flow in Compliant Arteries...

Time	Section A
10:30–10:50	Igor Pažanin* and Eduard Marušić–Paloka Fluid Flow Through a Helical Pipe
10:55–11:15	Bruce A. Watson The Darboux–Crum Transformation and Sturm–Liouville Problems with Eigenparameter Dependent Boundary Conditions
11:20–11:40	Andro Mikelić, Vincent Devigne* and C. J. van Duijn Taylor's Dispersion for Reactive Transport Model Using Asymptotic Methods
11:45–12:05	Thierry Clopeau* and Vincent Devigne Numerical Analysis of Flow, Transport and Chemical Processes in a Porous Medium
12:10–12:30	Dragan Jukić, Kristian Sabo* and Rudolf Scitovski Parameter Estimation Problem for Michaelis–Menten Model

Monday, June 20

Time	Section A
17:00–17:45	Andrey L. Piatnitski Gamma–Convergence of Nonconvex Variational Functionals Defined on a Random Checker Board Structure

Time	Section A
18:00–18:20	Marko Vrdoljak* and Nenad Antonić Gradient Methods for Multiple State Optimal Design Problems
18:25–18:45	Krešimir Burazin* and Nenad Antonić Computer-Aided Computation of Asymptotic Expansions in Nonlinear Homogenisation Problems with Memory Effects
18:50–19:10	Martin Lazar* and Nenad Antonić H-Measures Applied to Nonhyperbolic Equations
19:15–19:35	Olivier Bodart Asymptotic Approximation of the Solution of Stokes Equations in a Domain with Highly Oscillating Boundary

Tuesday, June 21

Time	**Section A** devoted to prof. Aganović
9:00– 9:45	Andro Mikelić Method of Homogenization Applied to Dispersion, Convection and Reaction in Porous Media

Time	**Section A** devoted to prof. Aganović
10:10–10:55	Krešimir Veselić On the Use of Sylvester and Lyapunov Equations in the Perturbation Estimates of Operators in Mathematical Physics
11:00–11:20	Ibrahim Aganović, Josip Tambača* and Zvonimir Tutek Reduction of Dimension in Micropolar Elasticity
11:25–11:45	Brahim Amaziane, Alaine Bourgeat and Mladen Jurak* Effective Transport of Solute in Heterogeneous Porous Media
11:50–12:10	Nermina Mujaković Nonhomogeneous Boundary Value Problem for One-Dimensional Compressible Viscous Micropolar Fluid Model
12:15–12:35	Josip Tambača and Igor Velčić* Evolution Model of Linearized Micropolar Plate

Tuesday, June 21

Time	Section A
17:00–17:45	Giovanni P. Galdi Some Geometric and Function–Analytic Properties of the Set of Steady Solutions to the Navier–Stokes Equations Past an Obstacle

Time	Section A	Section B
18:00–18:20	Boris Muha Existence of Periodic Solution of Fluid Flow Through a Pipe	Vedran Krčadinac* and Vladimir Volenec Computer Search for Identities in IM-quasigroups
18:25–18:45	Sonja Currie Dirichlet–Neumann Bracketing for Boundary Value Problems on Graphs	Vladimir Volenec and Zdenka Kolar–Begović* The Meaning of Computer Search in the Studying Some Classes of IM–quasigroups
18:50–19:10	Tina Bosner* and Mladen Rogina Numerically Stable Algorithm for Cycloidal Curves	Stefan Ratschan Constraint Based Verification of Hybrid Dynamical Systems
19:15–19:35	Zlatko Udovičić Choice of Trace in Wavelet Transform with Haar's and Daubechies Coefficients of Order 2	Andrei V. Prasolov On Cryptosystems Based on Polynomial Automorphisms

Wednesday, June 22

Time	Section A
9:00– 9:45	Alain Bourgeat Multiscale Modeling of an Underground Nuclear Waste Site

Time	Section A
10:10–10:30	Claudia Timofte Homogenization Results for Boundary Climatization Problems
10:35–10:55	Anders Holmbom* and Jeanette Silfver On the Convergence of Some Pairs of Weakly Convergent Sequences
11:00–11:20	Anders Holmbom, Nils Svanstedt* and Niklas Wellander Asymptotic Analysis of Parabolic Problems with Multiple Time and Spatial Scales
11:25–11:45	József Gáll* and Gyula Pap Random Field Forward Interest Rate Models and Market Price of Risk
11:50–12:10	József Gáll and Gyula Pap* Statistical Inference for Forward Interest Rate Models Driven by a Random Field

Morning session continued on the next page.

Wednesday, June 22

Time	Section A
12:15–12:35	Eugene B. Postnikov Evaluation of the Wavelet Transform with the Complex Morlet Basis via Solving the Cauchy Problem for the Diffusion Equations

Time	Section A
17:00–17:45	MariaRosaria Padula Viscous Incompressible Fluids in Domains with Elastic Walls

Time	Section A	Section B
18:00–18:20	Eduard Marušić–Paloka and Maja Starčević* Rigorous Derivation of Compressible Reynolds Equation for Gas Lubrication	Ioan Golet Approximation Theorems for Functions with Values in PN Spaces
18:25–18:45	Saša Krešić–Jurić The Heisenberg Magnet Equation and the Birkhoff Factorization	Andrei V. Prasolov Windows©Design for the Computer Algebra System Magma
18:50–19:10	Nela Bosner Increasing Efficiency of Inverse Iteration	Andrija Raguž On Equivalence of Some Minimization Problems on the Space of Sawtooth Functions
19:15–19:35	Luka Grubišić Saturation Assumptions for Ritz Value Approximation Methods	Gunter Neumann* and Stefan Schuster Continuous Model for the Rock–Scissors–Paper Game between the Bacteriocin Producing Bacteria

Thursday, June 23

Time	Section A
9:00– 9:45	Olivier Pironneau Sensitivity with Respect to Discontinous Data

Time	Section A
10:10–10:30	Luka Sopta, Senka Maćešić*, Danko Holjević, Nelida Črnjarić–Žic, Jerko Škifić, Siniša Družeta and Bojan Crnković Tribalj Dam–Break and Flood Wave Propagation
10:35–10:55	Nelida Črnjarić–Žic*, Senka Maćešić and Bojan Crnković Efficient Implementation of WENO Schemes to Nonuniform Meshes
11:00–11:20	Hervé Guillard Numerical Multiphase Modeling of Bubbly Flows
11:25–11:45	Ágnes Baran* and Gisbert Stoyan A Higher Order Non-Conforming Finite Element Family
11:50–12:10	Sándor Baran Prediction of Hungarian Mortality Rates Using Lee–Carter Method

Morning session continued on the next page.

Thursday, June 23

Time	Section A
12:15–12:35	Mervan Pašić Singular Behaviour of Bounded Radial Solutions of p-Laplacian

Excursion

Friday, June 24

Time	Section A
9:00– 9:20	Zlatko Drmač New LAPACK SVD Algorithm
9:25– 9:45	Saša Singer*, Emil Coffou and Mladen Rogina Accurate Computation of Gaussian Quadrature for Tension Powers

Time	Section A
10:10–10:30	Gašper Jaklič, Jernej Kozak, Marjeta Krajnc and Emil Žagar* Approximation of Circle Arcs by Parametric Polynomial Curves
10:35–10:55	Sanja Singer* and Mladen Rogina Conditions of Matrices in Discrete Tension Spline Approximations of DMBVP
11:00–11:20	Vjeran Hari Convergence of a Block-oriented Quasi-cyclic Jacobi Method
11:25–11:45	Miljenko Marušić* and Ivo Beroš A Difference Scheme for Singularly Perturbed Two-point Boundary Value Problems Based on Interpolation by Exponential Sum

Morning session continued on the next page.

Friday, June 24

Time	Section A
11:50–12:10	Daniel Kressner Product Eigenvalue Problems

INVITED LECTURES

WEDNESDAY, 9:00–9:45, SECTION A

Multiscale Modeling of an Underground Nuclear Waste Site

Alain Bourgeat

Abstract. An underground nuclear waste repository consist of nuclear waste packages stored in excavated vaults, all connected by drifts and galleries or tunnels which are backfilled after the packages storage. Usually, for safety reasons, the entire repository site is embedded in a low permeability layer. On the one hand, the model of a repository site should include the multiscale geometry, the large variations of the geology and the coupling of the different phenomena. But on the other hand far field simulations for performance assessments cannot use such detailed models. We will show how mathematical homogenization and asymptotical methods are able to provide scaled up but accurate and physical macroscopic model. The talks will address the main challenges in this field of up scaling: high number of leaking packages; a damaged zone, randomness of the packages content, boundary layers,...

MONDAY, 9:15–10:00, SECTION A

An Effective Model of Blood Flow in Compliant Arteries: Modeling, Analysis, Simulation and Experimental Validation

Sunčica Čanić*, Giovanna Guidoboni, Andro Mikelić and Josip Tambača

Abstract. Due to a tremendous complexity of the human cardiovascular system it remains unfeasible to numerically simulate larger sections of the circulatory system using the full three-dimensional equations for blood flow in compliant vessels. This is why simplified, effective equations are called for. In this vein, a variety of one-dimensional models have been widely used to study the flow of blood in axially symmetric sections of the vascular system. They are then coupled with the three-dimensional models when one-dimensional approximations are no longer appropriate. In all of the one-D models the typical question of closure related to averaging has been resolved by assuming an ad hoc closure in the form of a prescribed (estimated, experimentally calculated) axial velocity profile.

To avoid prescribing an ad hoc closure a novel system of closed effective equations is obtained. The novel closure technique uses ideas from homogenization

theory for porous media flows. The resulting two-dimensional ("essentially one-dimensional") system couples the incompressible viscous Navier–Stokes equations with the equations describing the vessel wall behavior. Vessel walls are described using two substantially different models: one is the linearly elastic membrane (and Koiter) shell model, and the other is a viscoelastic shell model with the viscoelasticity of the "Kelvin–Voight"-type.

The effective equations are in the form of a hyperbolic-parabolic system with "long-term" and "short-term" memory. The long-term memory, given as a convolution integral, explicitly captures the viscoelastic effects that a viscous fluid causes on the motion of an elastic structure. The short-term memory given as a time-derivative of the structure displacement, captures the leading contribution of the viscoelasticity of the structure.

Existence of a solution to a version of the reduced equations is obtained.

Numerical simulations of the solution corresponding to the actual patient data will be shown. A comparison with experiments performed at the Cardiovascular Research Laboratory at the Texas Heart Institute will be discussed. Ultrasound velocity and wall motion measurements, performed by Dr. Craig Hartley (Baylor College of Medicine), show excellent agreement with the numerically calculated solutions. The circulatory flow loop was assembled with help from Dr. Doreen Rosenstrauch (Texas Heart Institute) and undergraduate student Joy Chavez. Roderick McDonald grant from the Texas Heart Institute is acknowledged. This research has been supported by the National Science Foundation and the National Institutes for Health.

TUESDAY, 17:00–17:45, SECTION A

Some Geometric and Function–Analytic Properties of the Set of Steady Solutions to the Navier–Stokes Equations Past an Obstacle

Giovanni P. Galdi

Abstract. Consider a body B (compact set of $\mathbb{R}^3$) moving with constant velocity V in an incompressible Navier–Stokes fluid subject to an external force f and filling the whole space. No restrictions are imposed on f and V, other than f belonging to a suitable Lebesgue class L and V being nonzero. Let S denote the set of corresponding steady solutions to the Navier–Stokes equation with velocity field having a finite Dirichlet integral. In this talk we show that for any generic force f; in L and for any nonzero V, S is constituted by a finite (odd) number of solutions. Moreover, we show that when $f = 0$, S is homeomorphic to a compact set of $\mathbb{R}^N$, where N depends only on V, B and on the kinematic viscosity ν of the fluid. Finally, there exists a finite set, E_n, of points in $\mathbb{R}^n$ such that if two elements of S coincide in E_n, then they are identical. The integer n depends on V, B and ν as well.

TUESDAY, 9:00–9:45, SECTION A

Method of Homogenization Applied to Dispersion, Convection and Reaction in Porous Media

Andro Mikelić

Abstract. The homogenization technique, which is a rigorous method of averaging by multiple scale expansions, is applied to the transport of reacting solute in a porous medium. We focus our attention on situations when Peclet's and Damkohler's numbers are important. Starting from the pores, with the usual convection-diffusion equation for the solute and a first-order chemical reaction for the solute particles at the pore boundaries, we give a derivation of the 3D dispersion tensor for solute concentration. For particular pore geometry and particular scaling, we find the well-known Taylor dispersion formula. Our approach allows handling higher Peclet numbers than in the literature. For important Damkohler's number, our method gives important corrections in the upscaled transport and reactive terms.

WEDNESDAY, 17:00–17:45, SECTION A

Viscous Incompressible Fluids in Domains with Elastic Walls

MariaRosaria Padula

Abstract. In this note we present a global existence theorem of weak solutions to model equations governing interaction between a viscous incompressible fluid and an elastic structure, cf. [2], [3]. The fluid moves in a two dimensional layer, and the wall is an elastic line satisfying the so-called generalized string model.

One exact solution is the rest with a straight horizontal free surface. A previous result has been obtained by [1], where it has been proved an existence theorem of regular solutions, local in time, for small initial data. In [4] global existence in three dimensional bounded domain has been proved for a different model with different boundary conditions. Our principal result is global in time existence theorem when norms of initial data are less than an explicit upper bound.

The method arises by a substantial modification of classical Galerkin method.

The interest of our result is double because, first, we change the original initial value problem by deleting one initial condition, second, we construct a solution through the classical Galerkin method for which there have been constructed several computing codes.

The model is described by the following initial boundary value problem

$$\begin{aligned}
\nabla \cdot \mathbf{v} = 0, \qquad & x, y \in \Omega_t;\\
\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = \nu \nabla \mathbf{S}(\mathbf{v}) - \nabla p, \qquad & x, y \in \Omega_t;\\
-\partial_t^2 \eta + \beta \partial_x^2 \eta + \gamma \partial_t \partial_x^2 \eta - \alpha \partial_x^4 \eta - \sigma \eta = H(\eta), \qquad & x \in (0,1);\\
H(\eta) = -p + \nu \mathbf{n} \cdot \mathbf{S}(\mathbf{v}) \mathbf{n}, \qquad \mathbf{t} \cdot \mathbf{S}(\mathbf{v}) \mathbf{n} = 0 \qquad & \text{on} \quad \Gamma_t;\\
\partial_t \eta = (\mathbf{v} \cdot \tilde{\mathbf{n}})(x, \eta, t), \qquad & x, y \in \Gamma_t;\\
\mathbf{v}(x, 0, t) = 0, \qquad & x \in (0,1);\\
\mathbf{v}(x, y, 0) = \mathbf{v}_0(x, y), \qquad x, y \in \Omega_0, \quad \eta(x, 0) = \eta_0(x), \qquad & x \in (0,1).
\end{aligned} \tag{1}$$

The constants α, β, γ, σ characteristic of the elastic boundary are positive constants, ν is the kinematical viscosity, furthermore, $\mathbf{S}(\mathbf{v})$ denotes the symmetric part of the velocity gradient tensor $\mathbf{S}(\mathbf{v}) = \nabla \mathbf{v} + \nabla \mathbf{v}^T$.

On the part of the boundary $x = 0$, $x = 1$ we assume periodicity, precisely, for $u \in H^1(\Omega_t)$ with zero value at $y = 0$ and periodic in x, we write $u \in H^1_\sharp(\Omega_t)$, for $\eta \in H^2(0,1)$, periodic in x we write $\eta \in H^2_\sharp(0,1)$.

We prove the following existence theorem

Theorem Let $\mathbf{v}_0 \in H^1_\sharp(\Omega_0)$, $\eta_0 \in H^2_\sharp(0,1)$, and let

$$\int_{\Omega_0} v_0^2 \, dx \, dy + \int_0^1 \mathbf{v}_0 \cdot \tilde{\mathbf{n}}(\eta_0)]^2 \, dx$$
$$+ \beta \int_0^1 (\partial_x \eta_0)^2 \, dx + \alpha \int_0^1 (\partial_x^2 \eta_0)^2 \, dx + \sigma \int_0^1 \eta_0^2 \, dx < \frac{1}{4}.$$

Then, there exists a global solution of (1) with

$$\begin{aligned}
&\mathbf{v} \in L^\infty(0, T; L^2(\Omega_t)) \cap L^2(0, T; H^1_\sharp(\Omega_t)),\\
&\eta \in L^\infty(0, T; H^2_\sharp(0,1)),\\
&\eta_{,t} \in L^2(0, T; H^1_\sharp(0,1)) \cap (L^\infty(0, T; L^2_\sharp(0,1)).
\end{aligned}$$

This note is a preview of a article written in collaboration with Guidorzi, Plotnikov, in progress.

References

[1] H. Beirao Da Veiga, *On existence of strong solution to a coupled fluid structure evolution problem*, J. Math. Fluid Mech., 6 (2004), pp. 21–52.

[2] A. Quarteroni, M. Tuveri and A. Veneziani, *Computational vascular fluid dynamics: Problems, models and methods* Comput. Visual Sci., 2 (2000), pp. 163–197.

[3] L. Formaggia, J.–F. Gerbeau, F. Nobile, A. Quarteroni, *On the coupling of 3D and 1D NAvier–Stokes equations for flow problems in compliant vessels*, preprint INRIA no. N3862, 2000, also in Comput. Methods Appl. Mech. Engrg., 191 (2001), pp. 561–582.

[4] A. Chambolle, B. Desjardin, M. J. Esteban, C. Grandmont, *Existence of weak solutions for an unsteady fluid-plate interaction problem*, University Paris-Dauphine, preprint no. 0245, 2002.

MONDAY, 17:00–17:45, SECTION A

Gamma–Convergence of Nonconvex Variational Functionals Defined on a Random Checker Board Structure

Andrey L. Piatnitski

Abstract. The talk will focus on the asymptotic behaviour of random nonconvex variational functionals describing a discrete square grid made of small springs. We suppose that there are two types of springs in the grid, those which show the linear response to stretching, and those which do not resist the streching above a certain critical level.

Under the assumption that the random variables characterizing the choice of the type of spring at different locations are independent and identically distributed, we will describe the Gamma–limit of the said functionals.

THURSDAY, 9:00–9:45, SECTION A

Sensitivity with Respect to Discontinous Data

Olivier Pironneau

Abstract. We propose an extention of the calculus of variations so as to apply it in a formal maner even when the functions are non differentiable by using distribution derivatives. We will apply the method to the sensitivity of Darcy's hydrostatic pressure with respect to the position of discontinuities in the porosity and also to the sensitivity of Euler's equations for fluids. By this method, we will also analyze the sensitivity of shocks in compressible flows for an airfoil with respect to incidence angle.

Finally we will discuss the connection between optiml shape design and topological optimization.

TUESDAY, 10:10–10:55, SECTION A

On the Use of Sylvester and Lyapunov Equations in the Perturbation Estimates of Operators in Mathematical Physics

Krešimir Veselić

Abstract. Lyapunov and Sylevester equations, joined by the standard monotonicity arguments for eigenvalues, are able to give elegant and sometimes quite sharp estimates for the eigensolution of unbounded selfadjoint operators. Our particular attention wil be given to partial differential operators of Mathematical Physics, both classical and quantum.

Some of the presented results are a joint work with L. Grubišić, Hagen.

CONTRIBUTED TALKS

TUESDAY, 11:00–11:20, SECTION A

Reduction of Dimension in Micropolar Elasticity

Ibrahim Aganović, Josip Tambača* and Zvonimir Tutek

Abstract. In this work we derive a model of elastic rods from the three-dimensional linearized micropolar elasticity. Derivation is based on the asymptotic expansion method with respect to the thickness of the rod. The leading displacement is identified as the unique solution of a certain one-dimensional problem.

The main idea of this work is twofold. Firstly, we formulate the set of rigorous rules of the asymptotic expansion method called *Ansatz*. These rules allow us to derive the lower dimensional models without any *a priori* assumption on the scaling of the unknowns. Secondly, we apply the method on the three-dimensional linearized micropolar rod–like bodies. Micropolar elasticity is formulated in terms of two kinematic vector fields, displacements and microrotations of material points where material points are allowed to rotate without stretch. The obtained one-dimensional limit problem turns out to be of the Timoshenko type rod model. Namely, the leading microrotation is related to the macrorotiation of the cross-section and it appears as the unknown in the model. Through the appropriate convergence result we consider the limit one–dimensional model mathematically justified.

TUESDAY, 11:25–11:45, SECTION A

Effective Transport of Solute in Heterogeneous Porous Media

Brahim Amaziane, Alaine Bourgeat and Mladen Jurak*

Abstract. We consider a model of passive solute transport through highly heterogeneous porous media. The flow is governed by a coupled system of an elliptic equation and a linear convection-diffusion equation, with diffusion small with respect to the convection. We define a macroscale transport model by use of asymptotic expansion technique and perform numerical computations to verify correctness of macroscale model. Effective macroscale properties are calculated by finite volume method on structured grid, while fluid flow simulations are performed by mixed finite element method and vertex centered finite volume method on unstructured grid. The results of microscale and macroscale simulations are compared in terms of the first two spatial moments.

THURSDAY, 11:25–11:45, SECTION A

A Higher Order Non-Conforming Finite Element Family

Ágnes Baran* and Gisbert Stoyan

Abstract. We consider polynomial finite elements for the approximation of the Stokes problem which are continuous in the Gauss–Legendre points of the elements sides, i.e., generalize the classical Crouzeix–Raviart, Fortin–Soulie and Crouzeix–Falk elements. We show that, for any even order, by these Gauss–Legendre elements also the grid singularity of the well-known Scott–Vogelius elements is eliminated. The difference between Scott–Vogelius and Gauss–Legendre elements is just a non-conforming bubble on every triangle. The algebraic property of Scott–Vogelius elements to possess a Crouzeix–Velte decomposition (which is advantageous when solving the corresponding linear equations) carries over the Gauss–Legendre elements despite of those bubbles.

THURSDAY, 11:50–12:10, SECTION A

Prediction of Hungarian Mortality Rates Using Lee–Carter Method

Sándor Baran

Abstract. We applied a modified version the popular Lee–Carter method [1] for prediction of the mortality rates in Hungary for the period 2004–2040. The data are the mortality rates $m_{x,t}$ of age x at year t and the original Lee–Carter model is

$$\log(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}, \qquad x = 0, 1, \dots, N, \quad t = 1, 2, \dots, T,$$

where parameters a_x and b_x depend only on age while the mortality index k_t forms an ARIMA$(0,1,0)$ process with a constant and $\varepsilon_{x,t}$ is a random noise.

We considered separately men and women and we made predictions for ages 0–100 on the basis of years 1949–2003 and also using only data for the period 1989–2003. In the first case the fitted model is

$$\log(m_{x,t}) = a_x + b_x^{(1)} k_t^{(1)} + b_x^{(2)} k_t^{(2)} + b_x^{(3)} k_t^{(3)} + \varepsilon_{x,t},$$

where both for men and women $k_t^{(1)}$ is an ARIMA$(0,1,0)$ process with constant, $k_t^{(2)}$ is an ARIMA$(0,1,0)$ process without constant and $k_t^{(3)}$ is an AR(1) process. However, this model predicts increasing mortality rates for men of ages 45–55 because the fast decrease of the rates between 1992 and 2003 can not compensate the increase of period 1960–1991. This result also shows that the Lee–Carter method is hardly applicable for the countries where the mortality rates are so changing as in Hungary.

For the data of the last 15 years the fitted model for women is the original Lee–Carter model, i.e., $k_t^{(2)}$ and $k_t^{(3)}$ form white noise, while for the men $k_t^{(2)}$ is an AR(1) process and $k_t^{(3)}$ is again a white noise. In this case the mortality rates for all ages show a decreasing trend which is rather similar to the data from the U.S. and Australia [2] where the Lee–Carter method was successfully applied.

Research is supported by the ING Insurance Co. Ltd., Hungary. Joint work with József Gáll, Márton Ispány and Gyula Pap.

References

[1] R. D. Lee and L. R. Carter, *Modeling and forecasting the time series of U.S. mortality*, JASA, 87 (1992), pp. 659–671.

[2] H. Booth, J. Maindonald and L. Smith, *Age-time interactions in mortality projection: applying Lee–Carter to Australia*, Working Paper, The Australian National University, 2002.

MONDAY, 19:15–19:35, SECTION A

Asymptotic Approximation of the Solution of Stokes Equations in a Domain with Highly Oscillating Boundary

Olivier Bodart

Abstract. What we present is a part of a joint work with Y. Amirat, U. De Maio and A. Gaudiello. In ths presentation, we consider a viscous incompressible fluid filling an infinite horizontal domain limited at the bottom by a smooth wall $\mathcal{P}$ and at the top by a rough wall $\mathcal{R}_\varepsilon$. We assume that $\mathcal{P}$ moves at a constant horizontal velocity $g = (g', 0)$, $g' \in \mathbb{R}^2$, and that $\mathcal{R}_\varepsilon$ is at rest. The wall $\mathcal{R}_\varepsilon$ is assumed to consist of a plane wall covered with periodic asperities which size depends on a small parameter $\varepsilon > 0$, and with a fixed height. Let $0 < a_i < b_i < l_i$, $i = 1, 2$, $S = (0, l_1) \times (0, l_2)$, $\widetilde{S} = (a_1, b_1) \times (a_2, b_2)$, $S_\varepsilon = \varepsilon S$, $\widetilde{S}_\varepsilon = \varepsilon \widetilde{S}$, and let η_ε be the S_ε-periodic function defined (for the sake of simplicity) on S_ε by

$$\eta_\varepsilon(x') = \begin{cases} l_3 & \text{if } x' \in S_\varepsilon \setminus \widetilde{S}_\varepsilon, \\ l_3' & \text{if } x' \in \widetilde{S}_\varepsilon, \end{cases}$$

with $l_3' > l_3 > 0$, and $x' = (x_1, x_2$. Here, η_ε S_ε-periodic means that η_ε is periodic with respect to x_i with period εl_i, for $i = 1, 2$. The domain of the flow is

$$\mathcal{O}_\varepsilon = \{(x', x_3) \in \mathbb{R}^3 \ : \ x' \in \mathbb{R}^2, \ b(x') < x_3 < \eta_\varepsilon(x')\},$$

where b is a smooth function on $\mathbb{R}^2$, S-periodic (that is periodic with respect to x_i with period l_i, for $i = 1, 2$), such that $b(x') < l_3$ for all $x' \in \mathbb{R}^2$. It is bounded at the bottom by

$$\mathcal{P} = \{(x', x_3) \in \mathbb{R}^3 \ : \ x' \in \mathbb{R}^2, \ x_3 = b(x')\},$$

and at the top by $\mathcal{R}_\varepsilon = \partial\mathcal{O}_\varepsilon \setminus \mathcal{P}$, where $\partial\mathcal{O}_\varepsilon$ denotes the boundary of $\mathcal{O}_\varepsilon$. The profile of the asperities is comb shaped. The present work can be extended to more general functions η_ε.

The velocity $u_\varepsilon = (u_{\varepsilon 1}, u_{\varepsilon 2}, u_{\varepsilon 3})$ and the pressure p_ε of the fluid are assumes to satisfy the stationary Stokes equations.

We study the asymptotic behavior, as ε goes to 0, of $(u_\varepsilon, p_\varepsilon)$. The main difficulties arise from the fact that the height of the asperities does not vanish when ε tends to 0. We give a convergence result to a pair of functions (u_0, p_0) defined in the limit of the sequence of domains. Moreover, using an appropriate boundary layer corrector we derive and analyze an asymptotic approximation of the flow.

WEDNESDAY, 18:50–19:10, SECTION A

Increasing Efficiency of Inverse Iteration

Nela Bosner

Abstract. Inverse iteration is a simple and non-efficient method for computing few eigenvalues with minimal absolute value and corresponding eigenvectors of a symmetric matrix. The idea is to increase its efficiency by technique similar to multigrid methods used for solving linear systems. This approach is not new, but until now multigrid was mostly used for solving linear system which appear in Rayleigh quotient iteration, inverse iteration and related iterative methods. Instead of choosing appropriate coordinates (grids), our algorithm performs inverse iteration on a sequence of subspaces with decreasing dimensions (multispace). Block Lanczos method is used for the selection of a smaller subspace. This will produce a banded matrix, which makes inverse iteration even faster in the smaller dimensions. A convergence analysis is presented, together with numerical results.

TUESDAY, 18:50–19:10, SECTION A

Numerically Stable Algorithm for Cycloidal Curves

Tina Bosner* and Mladen Rogina

Abstract. We propose a knot insertion algorithm for splines that are piecewisely in $\mathrm{L}\{1, x, \sin x, \cos x\}$. Since an ECT-system on $[0, 2\pi]$ in this case does not exist, we construct a CCT-system by choosing the appropriate measures in the canonical representation. In this way, a B-basis can be constructed in much the same way as for weighted and tension splines. Thus we develop a corner cutting algorithm for lower order *cycloidal-curves*, though a straightforward generalization to higher order curves, where ECT-systems exist, is more complex. The important feature of the algorithm is high numerical stability and simple implementation.

MONDAY, 18:25–18:45, SECTION A

Computer-Aided Computation of Asymptotic Expansions in Nonlinear Homogenisation Problems with Memory Effects

Krešimir Burazin* and Nenad Antonić

Abstract. Homogenisation problems which introduce memory effects are difficult, and despite three decades of research, the available results are still restricted to particular types of equations. For nonlinear equations, only the method of asymptotic expansion shows some promise for success, in particular by a possibility to reduce complicated integrals to graphs.

We shall present algorithms developed to facilitate this manipulation, and produce both graphs and formulæ with multiple integrals needed in this research.

MONDAY, 11:45–12:05, SECTION A

Numerical Analysis of Flow, Transport and Chemical Processes in a Porous Medium

Thierry Clopeau* and Vincent Devigne

Abstract. In this talk we present numerical and analytical results on the reactive tranport of a solute through a porous medium. The solute is undergoing precipitation/dissolution at the pore boundaries. These chemical processes are described by the 1^{rst} order kinetics. The standard approach meet difficulties with over and under-saturation and consider the model introduced by C. J. van Duijn and al., where the dissolution rate is multivalued.

We consider the disretization of these convection-diffusion equations, coupled with the multivalued O.D.E. at the pore surfaces. Detailed numerical analysis of the problem is presented, together with a number of simulations.

TUESDAY, 18:25–18:45, SECTION A

Dirichlet–Neumann Bracketing for Boundary Value Problems on Graphs

Sonja Currie

Abstract. The spectral structure of second order boundary value problems on graphs is considered and a variational formulation is given. As a consequence we are able to formulate an analogue of Dirichlet–Neumann bracketing for boundary value problems on graphs. This in turn gives rise to eigenvalue and eigenfunction asymptotic approximations.

THURSDAY, 10:35–10:55, SECTION A

Efficient Implementation of WENO Schemes to Nonuniform Meshes

Nelida Črnjarić–Žic*, Senka Maćešić and Bojan Crnković

Abstract. Most of the standard papers about the WENO schemes consider their implementation to uniform meshes only. In that case the WENO reconstruction is preformed efficiently by using the algebraic expressions for evaluating the reconstruction values and the smoothness indicators from cell averages. The coefficients appearing in these expressions are constant, dependent just on the scheme order and not on the mesh size and the reconstruction function values, and can be found, for example, in Jiang, Shu, *Efficient implementation of WENO schemes*, J. Comp. Physics, 1996.

In problems where the geometrical properties must be taken into account or the solution has localized fine scale structure that must be resolved, it is computationally efficient to do local grid refinement, and therefore also desirable to have numerical schemes, which can be applied to nonuniform meshes. Finite volume WENO schemes extend naturally to nonuniform meshes although the reconstruction becomes quite complicated, depending on the complexity of the grid structure. In this paper we propose an efficient implementation of finite volume WENO schemes to nonuniform meshes. In order to save the computational cost in the nonuniform case, we suggest the way for precomputing the coefficients and linear weights for different orders of WENO schemes. Furthermore, for the smoothness indicators that are defined in an integral form we present the corresponding algebraic expressions in which the coefficients obtained as a linear combination of divided differences arise. In order to validate the new implementation, resulting schemes are applied in different test examples.

FRIDAY, 9:00–9:20, SECTION A

New LAPACK SVD Algorithm

Zlatko Drmač

Abstract. We present a new algorithm for SVD decomposition and its implementation in a robust and efficient mathematical software. The new algorithm is a combination of sophisticated preconditioning, preprocessing and a new variant of the one–sided Jacobi SVD algorithm. In terms of numerical reliability and capability of computing the SVD to high relative accuracy, it is superior to all other SVD algorithms, such as the QR or the divide and conquer algorithms (xGESVD and xGESDD from LAPACK). Moreover, in terms of efficiency, it outperforms the QR algorithm and it is comparable to the divide and conquer method.

The new algorithm will be included in the new release of the LAPACK library.

WEDNESDAY, 11:25–11:45, SECTION A

Random Field Forward Interest Rate Models and Market Price of Risk

József Gáll* and Gyula Pap

Abstract. There are several different methods in financial mathematics to construct models for forward interest rate curves of financial markets. Having such a model, bond processes, derivatives and other assets can be derived. Kennedy [5], Goldstein [3] and later Santa–Clara and Sornette [6], [7] proposed generalizations of the Heath–Jarrow–Morton (HJM) forward rate model (see [4]), in which the forward rate processes with different time-to-maturity value are not necessarily driven by the same (Wiener) process (unlike in the classical HJM case), i.e., for each $x \in \mathbb{R}_+$ we have

$$d_t f(t,x) = \alpha(t,x)dt + \sigma(t,x)d_t Z(t,x),$$

where $f(t,x)$ is the forward rate at time t with time to maturity x. (The two-time-parameter process Z is assumed to satisfy certain conditions.)

In the talk we consider the discrete time version of the model proposed by P. Santa–Clara and D. Sornette based on [1]. Given the necessary no-arbitrage conditions for such a market, we discuss the pricing of bond derivatives. We concentrate on special models where the driving field is a spatial AR sheets. We discuss the choice and the role of the market price of risk factors and the volatility parameter. We present some basic results concerning no-arbitrage conditions in such models and mention results on the maximum likelihood estimation of the volatility parameters based on [2].

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WEDNESDAY, 11:50–12:10, SECTION A

Statistical Inference for Forward Interest Rate Models Driven by a Random Field

József Gáll and Gyula Pap*

Abstract. There are several different methods in financial mathematics to construct models for forward interest rate curves of financial markets. Based on such a model, bond processes, (interest rate) derivatives can be priced. In the talk we shall study discrete time forward interest rate models driven by a random field (hence not a single process as in the Heath–Jarrow–Morton type models, see [5]), which were proposed in Gáll, Pap and Zuijlen [1], [2]. Our approach is motivated by Kennedy [6], Goldstein [4] and Santa–Clara and Sornette [7]. The model is of the form

$$f_{k+1,\ell} = f_{k,\ell} + \alpha_{k,\ell} + \beta(S_{k+1,\ell} - S_{k,\ell}),$$

where $f_{k,\ell}$ is the forward rate at time k with time to maturity ℓ, and $\{S_{k,\ell} \ : \ k, \ell \in \mathbb{Z}_+\}$ is an autoregressive Gaussian random field satisfying

$$S_{k,\ell} = S_{k,\ell-1} + \varrho S_{k-1,\ell} - \varrho S_{k-1,\ell-1} + \eta_{k,\ell},$$

where $\{\eta_{k,\ell} \ : \ k, \ell \in \mathbb{Z}_+\}$ are independent random variables with standard normal distribution. Introducing market price of risk and deriving no-arbitrage conditions, the drift $\alpha_{k,\ell}$ will be eliminated, and we turn to the maximum likelihood estimation of the parameters. Despite the lack of explicit solutions, we can show in several cases the (joint) asymptotic normality of the estimators. We can also prove strong consistency of the estimators in case of some parameters (like the volatility β). (Our earlier results on the volatility estimation can be found in [3].)

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WEDNESDAY, 18:00–18:20, SECTION B

Approximation Theorems for Functions with Values in PN Spaces

Ioan Golet

Abstract. Random functions have a special importance in the probability theory as well as in its applications. By regarding time series as random functions their predictability have increased. So, random functions have given important new tools in solving economics and engineering problems.

The notion of probabilistic normed spaces (briefly, PN space) was first defined by A. N. Šerstnev [2]. A PN space is a natural generalization of that of an ordinary normed linear space, in which the values of the norms are probability distribution functions rather than positive real numbers.

Alsina, Schweizer and Sklar in [1] gave a new and quite general definition of PN spaces. Many interesting results for these more general PN spaces have been recently achieved.

In this paper we consider a study of function with values in probabilistic normed spaces which also give an another enlargement of the notion of Šerstnev PN space. For this class of functions, topological properties and approximations theorems by polynomials are stated. Applications to the study of random functions (of random parameters) are also given.

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WEDNESDAY, 19:15–19:35, SECTION A

Saturation Assumptions for Ritz Value Approximation Methods

Luka Grubišić

Abstract. We are primarily concerned with an analysis of finite element methods for the eigenvalue/eigenvector problem for a selfadjoint elliptic operator.

A saturation assumption expresses and quantifies, through a saturation constant, the desired quality in any approximation method: Enlarged test space leads to better approximations. In particular, one defines — with a help of a saturation assumption — discrete *a posteriori* error estimates for elliptic boundary value problems which are not H^2 regular. This type of analysis is a particularly important step on a way towards an adaptive mesh refinement procedure. Only recently have Doerfler and Nochetto revealed a structure of such a saturation constant for a case of a boundary value problem.

We adapt and apply the analysis of Doerfler and Nochetto to an analysis of the eigenvalue problem by the means of the Ritz-vector residuum. We also derive a class of Temple–Kato eigenvalue estimates. The eigenvalue estimates are accompanied by a $\sin\Theta$-like result for the accompanying eigenvectors.

Our new residuum-based saturation constant will be compared with the saturation constant, featured in the Neymeyr's analysis of the Rayleigh–Ritz eigenvalue approximations. It will be shown that our discrete residuum estimate represents a first order estimate of the complete Ritz-vector residuum. This strongly corroborates the experimental results which were reported by Neymeyr.

At the end of the lecture we will present some numerical results to illustrate the developed theory.

THURSDAY, 11:00–11:20, SECTION A

Numerical Multiphase Modeling of Bubbly Flows

Hervé Guillard

Abstract. Bubbly flows appear in a large variety of engineering applications from the petroleum to the nuclear industry. A common model used in these contexts is the so-called drift-flux model where the slip velocity (the difference between the velocities of the gas and of the liquid) is expressed on the basis of empirical corelations. However, depending on these empirical corelations, these models are not always hyperbolic and this induces severe mathematical and numerical difficulties. Using asymptotic analysis in the limit of large drag terms, we propose an Eulerian mixture model where the slip velocity is expressed under the form of a Darcy like law. We study the mathematical properties of this model and describe a Godunov type scheme for its approximation. Some numerical relevant test-cases are presented.

FRIDAY, 11:00–11:20, SECTION A

Convergence of a Block-oriented Quasi-cyclic Jacobi Method

Vjeran Hari

Abstract. The global convergence of a special quasi-cyclic Jacobi method for computing the eigenvalue decomposition of a general symmetric matrix is proved. Besides, it is argued that the asymptotic convergence of the method is quadratic per quasi-sweep, even in the general case of multiple eigenvalues. The pivot strategy uses a general matrix block-partition. This strategy has been recently used by Drmač and Veselić in a paper on a remarkable efficiency improvement of the one-sided Jacobi method for computing the SVD of a general matrix.

WEDNESDAY, 10:35–10:55, SECTION A

On the Convergence of Some Pairs of Weakly Convergent Sequences

Anders Holmbom* and Jeanette Silfver

Abstract. Let us consider a sequence of integrals

$$\int_\Omega u^h(x)\cdot\nu^h(x)\phi(x)\,dx,\quad \phi\in D(\Omega),$$

where $\{u^h\}$ and $\{\nu^h\}$ are bounded in $L^2(\Omega)^N$. Under certain differential constraints on these sequences we will, up to a subsequence (u.t.s), arrive at a limit

$$\int_\Omega u(x)\cdot\nu(x)\phi(x)\,dx,$$

where u and ν are the corresponding weak $L^2(\Omega)^N$-limits. This phenomenon is known under the name compensated compactness. Alternatively, the sequence $\{\nu^h\}$ may appear as the result of a sequence $\{\tau^h\}$ of transforms acting on an admissible space $X\subset L^2(\Omega\times Y)^N$ of test functions v, where Y is the unit cube in $\mathbb{R}^N$. This approach provides us with a different kind of limit, a so called two-scale limit

$$\int_\Omega\int_Y u_0(x,y)\cdot v(x,y)\phi(x)\,dy\,dx,$$

where $u_0\in L^2(\Omega\times Y)^N$. Usually, these transforms are chosen such that

$$\nu^h(x)=\tau^h v(x)=v\left(x,\frac{x}{\varepsilon_h}\right),$$

where $\varepsilon_h\to 0$ when $h\to\infty$ and X consists of sufficiently smooth functions periodic in their second argument. This kind of convergence is named (periodic) two-scale convergence and appears u.t.s for any sequence $\{u^h\}$ bounded in $L^2(\Omega)^N$. However, such limits can be obtained also for other choices of $\{\tau^h\}$ and X if for some positive constant C independent of h and all $v\in X$

$$\lim_{h\to\infty}\|\tau^h v\|_{L^2(\Omega)^N}\le C\|v\|_{L^2(\Omega\times Y)^N}\quad\text{and}\quad\|\tau^h v\|_{L^2(\Omega)^N}\le C\|v\|_X.$$

We discuss the relationship between the two kinds of limits introduced above and the connection between two-scale limits and usual weak limits for some choices of $\{\tau^h\}$, where no periodicity assumptions are involved.

WEDNESDAY, 11:00–11:20, SECTION A

Asymptotic Analysis of Parabolic Problems with Multiple Time and Spatial Scales

Anders Holmbom, Nils Svanstedt* and Niklas Wellander

Abstract. We consider divergence structure parabolic problems of the form

$$\begin{aligned} \partial_t u^\varepsilon(x,t) - \nabla\cdot\left(a\left(x,\frac{x}{\varepsilon},\frac{x}{\varepsilon^2},t,\frac{t}{\varepsilon^k}\right)\nabla u^\varepsilon(x,t)\right) &= f(x,t), \quad && x\in\Omega, \quad && 0<t\le T,\\ u^\varepsilon(x,t) &= 0, && x\in\partial\Omega, && 0<t\le T,\\ u^\varepsilon(x,0) &= u_0(x). \end{aligned}$$

where the function $a(x,y,z,t,s)$ is oscillating periodically in (y,z) in space and in s in time. We use multiscale convergence theory to show that under standard boundedness and coercivity assumptions on a the sequence of solutions u^ε converges weakly in $L^2(0,T;H_0^1(\Omega))$ to the solution u to a homogenized problem

$$\begin{aligned} \partial_t u(x,t) - \nabla\cdot(b(x,t)\nabla u(x,t)) &= f(x,t), \quad && x\in\Omega, \quad && 0<t\le T,\\ u(x,t) &= 0, && x\in\partial\Omega, && 0<t\le T,\\ u(x,0) &= u_0(x). \end{aligned}$$

FRIDAY, 10:10–10:30, SECTION A

Approximation of Circle Arcs by Parametric Polynomial Curves

Gašper Jaklič, Jernej Kozak, Marjeta Krajnc and Emil Žagar*

Abstract. The approximation of circle arcs has been an important task in the Computer Aided Geometric Design (CAGD), Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM) for quite a while. Thought any circle arc could be exactly represented by a quadratic Bézier rational curve, some CAD/CAM systems require the polynomial representation of circle segments. Also, some important algorithms like lofting and blending are difficult to generalize to rational form. Among others, Lyche and Mørken have studied the problem of approximation of circle segments by polynomial parametric curves. They have found an excellent explicit approximation with the odd degree parametric polynomial curves, but conjectured that the even degree could be a tough task. Here, the general degree $n = 2^k(2\ell - 1)$ approximation is provided. Each pair $k \in \mathbb{N}_0$, $\ell \in \mathbb{N}$ determines a closed form approximating parametric curve based upon the Chebyshev polynomials of the first

and the second kind. The numerical evidence supplemented confirms that a circle segment can be approximated by this curve with radial error bounded by const h^{2n}, where h is the length of a segment.

MONDAY, 12:10–12:30, SECTION A

Parameter Estimation Problem for Michaelis–Menten Model

Dragan Jukić, Kristian Sabo* and Rudolf Scitovski

Abstract. In this paper we consider the ordinary least squares (OLS) and total least squares (TLS) problems for a Michaelis–Menten enzyme kinetic model

$$f(x; a, b) = \frac{ax}{b + x}, \quad a, b > 0.$$

This model is widely used in biochemistry, pharmacology, biology and medical research.

We show that both OLS and TLS problems for Michaelis–Menten model have a solution if the data (p_i, x_i, y_i), $i = 1, \ldots, m$, $m \geq 3$, satisfy some natural conditions. Finnaly we give several numerical examples.

TUESDAY, 18:00–18:20, SECTION B

Computer Search for Identities in IM-quasigroups

Vedran Krčadinac* and Vladimir Volenec

Abstract. An *IM-quasigroup* is a cancellative, solvable groupoid satisfying the idempotency and mediality laws: $aa = a$ and $ab \cdot cd = ac \cdot bd$. A standard model are the complex numbers with multiplication defined by $a \cdot b = (1 - q)a + qb$, for any $q \in \mathbb{C} \setminus \{0, 1\}$. Of particular interest are IM-quasigroups satisfying additional identities, such as hexagonal and GS-quasigroups. An identity will be called *admissible* provided there is at least one example within the standard model, i.e., a binary operation on the complex numbers defined by the aforementioned formula for at least one $q \in \mathbb{C} \setminus \{0, 1\}$. Geometric concepts such as midpoints, vectors and parallelograms can be introduced in any IM-quasigroup. Given three points a, b and c, there is a unique point d forming a parallelogram (a, b, c, d). Some IM-quasigroups, including hexagonal and GS-quasigroups, allow an explicit formula for d as a function of a, b and c.

We report on a systematic computer search for admissible identities in IM-quasigroups. A computer search for IM-quasigroups allowing a short formula for the fourth vertex of a parallelogram was also preformed. Besides the well-known

and well-studied types, other interesting classes of IM-quasigroups are discovered. In particular, a class remarkably similar to the GS-quasigroups was found. These quasigrups are related to a generalization of the golden ratio.

WEDNESDAY, 18:25–18:45, SECTION A

The Heisenberg Magnet Equation and the Birkhoff Factorization

Saša Krešić–Jurić

Abstract. A geometrical description of the Heisenberg magnet equation with classical spins is given in terms of flows on the quotient space G/H_+ where G is an infinite dimensional Lie group and H_+ is a subgroup of G. It is shown that the Heisenberg magnet equation can be integrated by solving a Riemann-Hilbert type of problem on G. It is also shown that the gauge transformation between the Heisenberg magnet and nonlinear Schrödinger equations can be interpreted as a map between a canonical pair of Birkhoff factorizations of G. For a special class of flows on G/H_+ a simple algebraic relation between soliton solutions of the nonlinear Schrödinger and Heisenberg magnet equations is derived.

FRIDAY, 11:50–12:10, SECTION A

Product Eigenvalue Problems

Daniel Kressner

Abstract. The *product eigenvalue problem* is concerned with computing eigenvalues and invariant subspaces of a matrix product

$$\Pi = A_p \cdot A_{p-1} \cdots A_1$$

for some matrices $A_1, \ldots, A_p$. It is of interest in several applications, in particular those related to physical and chemical processes that exhibit seasonal or periodic behaviour. Other applications include optimal control problems, model reduction, bifurcation analysis and the computation of Floquet multipliers for partial differential equations. Working on the factors of Π instead of the explicitly formed product has several important advantages, e.g., preservation of physically meaningful properties and higher accuracy in the computed eigenvalues. In this talk, we will summarize recent developments on the theory, algorithms and software for product eigenvalue problems, as well as a prospectus for future work in this area. This is ongoing joint work with Zlatko Drmač, University of Zagreb, and Robert Granat and Bo Kågström, Umeå University, in the framework of an Emmy–Noether-fellowship from the DFG.

MONDAY, 18:50–19:10, SECTION A

H-Measures Applied to Nonhyperbolic Equations

Martin Lazar* and Nenad Antonić

Abstract. Since their introduction, H-measures have been mostly used in problems related to hyperbolic equations and systems. In this study we give an attempt to apply the H-masure theory to parabolic equations. Through a number of examples we try to present how the differences between parabolicity and hyperbolicity reflect in the theory.

We shall try to give some hints how to apply these results to equations that change their type, as the Tricomi equation.

FRIDAY, 11:25–11:45, SECTION A

A Difference Scheme for Singularly Perturbed Two-point Boundary Value Problems Based on Interpolation by Exponential Sum

Miljenko Marušić* and Ivo Beroš

Abstract. The difference scheme is used as a numerical approximation to the solution of a singularly perturbed two-point boundary value problem: $\varepsilon y''+by'+cy = f$. The difference scheme is derived from interpolation formulae for exponential sum. Convergence of the difference scheme is proved using interpolation error and limit properties of exponential sums.

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WEDNESDAY, 18:00–18:20, SECTION A

Rigorous Derivation of Compressible Reynolds Equation for Gas Lubrication

Eduard Marušić–Paloka and Maja Starčević*

Abstract. We give a rigorous justification of the compressible Reynolds model for gas lubrication, via asymptotic analysis. We start from the compressible Stokes system in a thin domain and study the limit as the domain thickness tends to zero. At the limit we find the known engineering model. The key of the proof is the strong convergence for the pressure obtained by its decomposition.

MONDAY, 11:20–11:40, SECTION A

Taylor's Dispersion for Reactive Transport Model Using Asymptotic Methods

Andro Mikelić, Vincent Devigne* and C. J. van Duijn

Abstract. Starting from rigorous results given by the theory we propose to illustrate the results on effective reactive transport through a pore by numerical samples and simulations. The talk is organized as follow. First, we recall briefly the background and assumptions of the model, the original case considered by Sir Taylor and introduce its dimensionless form. Then, we present the rigorously homogenized problem and compare it with the equation obtained by direct averaging. The coefficients turn to differ significantly for large Peclet and Damkohler's number. We give the analytical solution of the 1D-parabolic problems, the numerical methods employed in order to solve correctly the 2D-problem attached to the pore geometry and quantify numerically the error estimates with respect to the small parameter ϵ.

TUESDAY, 18:00–18:20, SECTION A

Existence of Periodic Solution of Fluid Flow Through a Pipe

Boris Muha

Abstract. A fluid flow through a pipe of finite length and variable cross-section is considered. Assuming periodic (in time) boundary conditions on pipe ends, existence of a periodic solution of corresponding Navier–Stokes system is proved for sufficiently thin pipes. The proof is based on asymptotic behaviour of the Navier–Stokes flow with respect to the pipe thickness.

TUESDAY, 11:50–12:10, SECTION A

Nonhomogeneous Boundary Value Problem for One-Dimensional Compressible Viscous Micropolar Fluid Model

Nermina Mujaković

Abstract. We consider nonstationary 1-D flow of a micropolar viscous compressible fluid, which is in a thermodynamic sense perfect and polytropic. It assumes that the boundary conditions for the velocity and microrotation are nonhomogeneous. Introducing the auxiliary functions we transform this problem in the problem with homogeneous boundary conditions, which we solve by the Faedo–Galerkin method and get a local-in-time existence of solution.

WEDNESDAY, 19:15–19:35, SECTION B

Continuous Model for the Rock–Scissors–Paper Game between the Bacteriocin Producing Bacteria

Gunter Neumann* and Stefan Schuster

Abstract. In this work several aspects of bacteriocin producing bacteria and their interplay is elucidated. Various attempts to model the Resistant, Producer and Sensitive E Coli strains in the so called RSP–Game (Rock–Scissors–Paper) have been made and arose the question, if there is a continuous model, which admits a cyclic structure like in fed batch cultures. The observations in experiments showed cyclic structure in spatial distribution of these three competing species, while in mice cultures migration seemed to be essential for the reinfection in the RSP-cycle. This paper tries to describe the possible models and to rule out others, to clarify the underlying dynamics. In Szabo [4] statistical effects (migration/mutation) seemed to be the driving force of the cycle, also in Kirkup and Riley [1] spatial effects enable periodicity. In Chemostat no periodic behavior was observed. On the other side fed batch systems can be constructed, which admit periodicity. The well known May–Leonard systems [3] admit a Hopf bifurcation but it is degenerate and hence inadequate. Also the traditional RSP-Game has no real counterpart. A parameter set was obtained for which a stable limit-cycle exists in the same Zeeman-class [2] admitting a full cycle in the three-species game. These parameters are in accordance with the observed relations of the E Coli strains. Further studies will be made to analyze the stability of this cycle to get the representativity of the available space for which periodicity is possible.

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THURSDAY, 12:15–12:35, SECTION A

Singular Behaviour of Bounded Radial Solutions of p-Laplacian

Mervan Pašić

Abstract. Bounded continuous radial solutions are studied for a class of p-Laplacian equation in a ball $B_R \subseteq R^N$. It is supposed that the nonlinear term in the equation is singular and rapid sign-changing near the boundary ∂B_R. In this case, the lower bounds for the Minkowski–Bouligand (box-counting) dimension of the graphs of any solution and its gradient are given in order to describe a kind of high concentration of the graphs of solutions near ∂B_R. Furthermore, the order of growth for singular boundary behaviour of the Lp norm of the gradient of solutions is derived. Finally, the existence of at least one solution satisfying obtained properties is shown.

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MONDAY, 10:30–10:50, SECTION A

Fluid Flow Through a Helical Pipe

Igor Pažanin* and Eduard Marušić–Paloka

Abstract. Fluid flow through helical pipes appears naturally in many applications, industrial as well as biological. In nature it can be found in blood vessels, in trachea or in water ducts for plants. Engineering applications cover a large number of devices such as pipelines, heat exchangers (Liebig cooler, cooling channel in the nozzle of a rocket engine), exhaust gas ducts of engines, chemical reactors, particle separators used in the mineral processing industry, coiled capillaries in viscometry etc.

We consider the flow of incompressible Newtonian fluid through a helical pipe governed by a pressure drop between the pipe's ends. When the problem is written in an adimensional form, suitable for an asymptotic analysis, three important geometric parameters appear: pipe's thickness, distance between two coils of the helix (helix step) and the helix diameter. We suppose that the pipe's thickness and the helix step have the same small order $\varepsilon \ll 1$ while the diameter of the helix is larger, of order 1. The existence and uniqueness of the solution for the corresponding boundary value problem for the Navier–Stokes system is discussed. Using the asymptotic analysis with respect to the small parameter ε, the effective behavior of the flow is found. The rigorous mathematical justification of the formally derived asymptotic approximation is given.

WEDNESDAY, 12:15–12:35, SECTION A

Evaluation of the Wavelet Transform with the Complex Morlet Basis via Solving the Cauchy Problem for the Diffusion Equations

Eugene B. Postnikov

Abstract. Let us consider the continuous transform

$$w(a,b) = \int_{-\infty}^{\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)\frac{dt}{\sqrt{a}}, \quad \psi(\xi) = \frac{1}{\sqrt[4]{\pi}}\left(e^{-i\omega_0\xi} - e^{-\frac{\omega_0^2}{2}}\right)e^{-\frac{\xi^2}{2}}.$$

where the asterisk means the complex conjugation and $\psi(\xi)$ is the complex Morlet basis. The usage of this wavelet has an advantage for the the signal processing because the result plays the role of a local spectrum.

The main result of this work is the following. It is shown that the wavelet transform with the Morlet basis can be easily performed if we replace the integration of the fast-oscillation function by the solution of the Cauchy problem for the

following diffusion differential equations:

$$
\begin{aligned}
\frac{\partial u_1}{\partial a} &= a\frac{\partial^2 u_1}{\partial b^2} + \omega_0\frac{\partial v_1}{\partial b},\\
\frac{\partial v_1}{\partial a} &= a\frac{\partial^2 v_1}{\partial b^2} - \omega_0\frac{\partial u_1}{\partial b},\\
\frac{\partial u_2}{\partial a} &= a\frac{\partial^2 u_2}{\partial b^2}.
\end{aligned}
$$

The initial values are

$$u_1(0,b) = u_2(0,b) = f(b)\sqrt[4]{4\pi}e^{-\omega_0^2/2}, \quad v_1(0,b) = 0.$$

Using this function, the requared wavelet transform is

$$w(a,b) = a^{-\frac{1}{2}}(u_1(a,b) - u_2(a,b) + iv_1(a,b)).$$

To solve the above equations there are robust methods, even on the non-uniform grid (The one of them is realized in the standard METLAB kernel.). It is a main advantage of the proposed algorithm. During the calculation, the time variable in the diffusion equation is associated with the scale variable of the wavelet transform. Therefore, the choice of a small enough step allows us to trace the evolution of the instant period in detail.

The results are illustrated by the application of this method to the processing of test examples, the Saturn rings' images obtained from the Cassini spacecraft, and the DNA sequences.

WEDNESDAY, 18:25–18:45, SECTION B

Windows© Design for the Computer Algebra System Magma

Andrei V. Prasolov

Abstract. The computer algebra system *Magma* was created by the Symbolic Algebra group at the University of Sydney in Australia:
`http://magma.maths.usyd.edu.au/magma/`

The system has proven to be a very efficient instrument of computation in many areas of modern algebra. However, the system has a relatively poor console design. Having started Magma, one sees one input/output window. Both input and output are plain texts, without rich text attributes like boldface or color. Other windows possibilities like Copy/Paste, mouse clicks/double clicks etc. are missing or significantly reduced.

We made an attempt to build a *Windows© emulator* for Magma, *WinMagma.exe*. It is a Win32 program. The program was designed and compiled using Borland© C++ Builder© 5.

See the pictures at `http://www.math.uit.no/users/andreip/Magma/`.

Picture 1 shows the starting window of WinMagma. The user can start Magma from within WinMagma, but the Magma window remains invisible (Picture 2). The user types then in the input box, and WinMagma sends the strokes to Magma. It catches then Magma's output and writes it to the output box. Both input and output are written to the history box. The forth box can be used as a rich text editor. The boxes are resizable, see Picture 3. The attributes of the text in all four boxes are enough flexible, see Picture 4. One can also save/load the content of each box, see Picture 5.

This windows emulator of Magma is only the first attempt of improving the design of that powerful computer algebra system. However, other useful functions can be easily built into this program *WinMagma.exe*.

TUESDAY, 19:15–19:35, SECTION B

On Cryptosystems Based on Polynomial Automorphisms

Andrei V. Prasolov

Abstract. We propose a new cryptographic scheme of El Gamal type.

Let K be a finite field with q elements, and let $R = K[X_1, X_2, \ldots, X_n]$ be the polynomial ring of n indeterminates. Consider polynomial mappings $K^n \to K^n$ given by n-tuples of polynomials $[F_1, F_2, \ldots, F_n]$, $F_i \in R$. Let $Map_0(n, K)$ be the set of **pure** polynomial mappings, and $Map(n, K)$ be the set of **reduced** mappings. The difference between the two is the following: the two mappings $F = [F_1, F_2, \ldots, F_n]$ and $G = [G_1, G_2, \ldots, G_n]$ are considered equal pure mappings iff $F_i = G_i$ for all i. The same two mappings are considered equal reduced mappings iff they define the same (set-theoretic) mapping $K^n \to K^n$. Let $F \circ G$ denote the composition of F and G. Let further $Aut_0(n, K)$ (respectively $Aut(n, K)$) denote the group of invertible elements of $Map_0(n, K)$ (respectively $Map_0(n, K)$) with respect to the operation $\circ$. Note that both $Map_0(n, K)$ and $Aut_0(n, K)$ are infinite sets, while $|Map(n, K)| = q^{q^n}$ and $|Aut(n, K)| = q^n!$ (actually, $Aut(n, K) \approx S_{q^n}$).

The scheme below can be based on either of the two sets. Let $M = Map_0(n, K)$ and $G = Aut_0(n, K)$, or $M = Map(n, K)$ and $G = Aut(n, K)$. Let r be a prime, and let $L : K^n \to K^n$ be a linear mapping of order r. Such mapping can be found for any r which divides $q^n - 1$. Let T be a **tame** polynomial mapping (product of elementary polynomial automorphisms, and affine automorphisms). Let s be a random integer, let $a = T^{-1} \circ L \circ T \in G$, and let $b = a^s \in G$. The 4-tuple (M, G, a, b) forms the **public key** for the proposed scheme.

Fix an integer d. Let $V_d \subseteq M$ be the K-linear subspace of mappings of degree $\leq d$. Actually, we can choose any other linear K-subspace $V \subseteq M$. A message to be encoded can be considered as a sequence of elements of K. Divide the message into blocks of length $\dim(V)$. Any such block corresponds to an element X_i of V. The elements X_i are encoded as follows (three variants). The sender chooses a random integer k, and sends the pair $(Y, Z) := (a^k, b^k \circ X_i)$, or $(Y, Z) := (a^k, X_i \circ b^k)$, or $(Y, Z) := (a^k, b^k + X_i)$.

It can be easily seen that the receiver can decode the message, having the **secret keys** s and r in his disposal: $X_i = Y^{r-s} \circ Z$, or $X_i = Z \circ Y^{r-s}$, or $X_i = Z - Y^s$.

WEDNESDAY, 18:50–19:10, SECTION B

On Equivalence of Some Minimization Problems on the Space of Sawtooth Functions

Andrija Raguž

Abstract. We formulate a problem of comparison of minima of some lower-semicontinuous functionals of Modica–Mortola type which aredefined on the space of sawtooth functions. We obtain somepreliminary results in this respect. We also indicate connections with asymptotic analysis for a class of functionals of Ginzburg–Landau type in one dimension in the case when minimizers exhibit oscillations on both externally and internally created small scale of order $O(\varepsilon^{1/3})$.

TUESDAY, 18:50–19:10, SECTION B

Constraint Based Verification of Hybrid Dynamical Systems

Stefan Ratschan

Abstract. A hybrid system is a dynamical system that has a state-space with both a continuous and a discrete component, where both components can influence each other. Such systems occur naturally as models of (discrete) computing devices that are embedded into a (continuous) physical environment. Here one wants to verify that a given hybrid system fulfills a desired property, for example, that it always stays within a certain set of safe states, or that it eventually reaches a set of target states.

The talk we be an overview on our work in this area. The approach formulates conditions on the evolution of trajectories as constraints in the first-order theory of the real numbers, and then applies a constraint-propagation based solver on these constraints. In the discrete-time case, although the problem is undecidable, we have an algorithm for which we could prove termination of the resulting algorithm for all inputs except for numerically ill-posed ones. In the continuous-time case we have an algorithm with a practically efficient implementation.

FRIDAY, 10:35–10:55, SECTION A

Conditions of Matrices in Discrete Tension Spline Approximations of DMBVP

Sanja Singer* and Mladen Rogina

Abstract. Some splines can be defined as solutions of differential multi point boundary value problems (DMBVP). In the numerical treatment of DMBVP, the differential operator is discretized by finite differences. We consider one dimensional discrete hyperbolic tension spline introduced in [1], where this approach leads to specially-structured pentadiagonal linear system.

Errors in exact methods for the solution of this linear system depends on condition numbers of corresponding matrices. If the chosen mesh is uniform, the system matrix is symmetric and positive definite, and it is easy to compute both, lower and upper bound, for its condition. In the more interesting non-uniform case, matrix is not symmetric, but in some circumstances we can nevertheless find an upper bound of its condition number.

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FRIDAY, 9:25–9:45, SECTION A

Accurate Computation of Gaussian Quadrature for Tension Powers

Saša Singer*, Emil Coffou and Mladen Rogina

Abstract. We consider Gaussian quadrature formulæ which exactly integrate a system of tension powers

$$1, x, x^2, \ldots, x^{n-3}, \sinh(px), \cosh(px),$$

on $[a, b]$, where $n \geq 4$ is an even integer and $p > 0$ is a given tension parameter. The existence and uniqueness of such formulæ follows from the Chebyshev space theory.

In some applications we need an efficient "on-demand" algorithm, that calculates the nodes and weights for a given value of p, at least for several low values of n (say $n \leq 20$). Unfortunately, no single approach can numerically achieve the required machine accuracy for all tension parameters of interest. By exploiting various

analytic techniques, translation invariance and known behaviour of tension powers and Gaussian quadrature formulæ in the limiting cases $p \to 0$ and $p \to \infty$, we can construct efficient and accurate algorithms for various ranges of p.

We will present an outline of the algorithm, followed by a detailed description of implementation of several key steps, with emphasis on numerical stability. The gains in accuracy of this approach will be illustrated by a few numerical examples.

THURSDAY, 10:10–10:30, SECTION A

Tribalj Dam–Break and Flood Wave Propagation

Luka Sopta, Senka Maćešić*, Danko Holjević, Nelida Črnjarić–Žic,
Jerko Škifić, Siniša Družeta and Bojan Crnković

Abstract. In this paper we present numerical simulations for the dam break flood wave propagation from Tribalj accumulation to Crikvenica (Croatia). The mathematical models we used were the one and the two-dimensional shallow-water equations. They were solved with the well-balanced finite volume numerical schemes which additionally include special numerical treatment of the wetting/drying front boundary. These schemes were tested on CADAM test problems.

The aim of this study was to assess potential damage in the cities of Tribalj and Crikvenica. Results of these simulations where used as the basis for urban planning and micro-zoning of the flood-risk areas. Several different dam break scenarios where considered, ranging from sudden dam disappearance to partial and dynamic breach formation. The research showed some advantages of numerical simulation over physical models.

TUESDAY, 12:15–12:35, SECTION A

Evolution Model of Linearized Micropolar Plate

Josip Tambača and Igor Velčić*

Abstract. In this paper we derive and mathematically justify a twodimensional evolution model and associated eigenvalue problem of micropolar plates starting from the linearized micropolar elasticity. Derivation is based on the asymptotic tehniques letting the thickness of the body tends to zero. The limit microrotation is then related to the macrorotation of the cross-section and the model is rewritten in macroscopic unknowns. The obtained model is then recognized as Reissner–Mindlin evolution plate model.

WEDNESDAY, 10:10–10:30, SECTION A

Homogenization Results for Boundary Climatization Problems

Claudia Timofte

Abstract. This talk deals with the homogenization of a nonlinear model for heat conduction through the exterior of a domain containing periodically distributed conductive grains. We assume that on the walls of the grains we have climatizators governing the heat flux through the boundary. The effective behavior of this nonlinear flow is described by a new elliptic boundary-value problem containing an extra zero-order term which captures the effect of the boundary climatization.

TUESDAY, 19:15–19:35, SECTION A

Choice of Trace in Wavelet Transform with Haar's and Daubechies Coefficients of Order 2

Zlatko Udovičić

Abstract. Most coefficients in wavelet transform of enough smooth function, in case when wavelets has enough number of vanishing moments, is negligible small. This fact has very important practical application. So, if we take wavelet transform of discreet function (we suposed that coefficients of transform are computed by using Mallat's pyramidal algorithm) and all coefficients which are by its absolute value less than given trace replace by zeroes (so called trasholding technics) length of wavelet transform will be significantly less than length of given function. Trashoding process is not reversible since some data will disappear forever. Of course, if used trace is bigger, the relative error of reconstructed function will be bigger.

In this paper we consider choice of trace in dependance of allowed relative error of reconstructed function. The coefficients of corresponding dilatation equation are Haar's and Daubechies coefficients of order 2. We used some geometric interpretation of pyramidal algorithm and basic results of theory of probability. By using deduced results we also determined optimal trace (minimization of relative error and maximization of number of coefficients which will be replaced by zeroes at the same time). At the end of paper, some examples are given which justify use of deduced results in practice.

TUESDAY, 18:25–18:45, SECTION B

The Meaning of Computer Search in the Studying Some Classes of IM–quasigroups

Vladimir Volenec and Zdenka Kolar–Begović*

Abstract. A computer search allowing the discovery some special values for the complex number $q \in \mathbb{C} \setminus \{0,1\}$ such that the quasigroup $C(q)$ is characterized by means of simple algebraic identities, which can be seen in the presentation of V. Krčadinac and V. Volenec. In this way we can find out simple formulae for defining the quaternary relation Par in such a quasigroup, where $\mathrm{Par}(a,b,c,d)$ means that a,b,c,d are the vertices of a parallelogram in this cyclical order.

We are going to report on the IM–quasigroup $C(q)$ for $q = \frac{1}{2}(1+\sqrt{5})$ the so called GS–quasigroup which has nice geometric representation which justifies the research of this quasigroup. In general GS–quasigroup the relation Par can be defined by means of several equivalent formulae which can be discovered in $C(q)$ for $q = \frac{1}{2}(1+\sqrt{5})$. We will choose such a formula for Par which allows the definition of the addition in general GS–quasigroup by the equivalency $c = a + b \Leftrightarrow \mathrm{Par}(0,a,c,b)$ where 0 is the chosen element in GS–quasigroup. The theorem about characterization of GS–quasigroup will be proved by means of the above mentioned formula.

MONDAY, 18:00–18:20, SECTION A

Gradient Methods for Multiple State Optimal Design Problems

Marko Vrdoljak* and Nenad Antonić

Abstract. We optimise a distribution of two isotropic materials that occupy a given body in $\mathbb{R}^3$. The optimality is described by the integral functional (cost) of temperatures u_1 and u_2 of the body obtained for two different source terms f_1 and f_2 with homogeneous Dirichlet boundary conditions.

The relaxation of this optimal design problem with multiple state equations is needed, introducing the notion of composite materials as fine mixtures of different phases, mathematically described by the homogenisation theory. The necessary conditions of optimality are derived via the Gâteaux derivative of cost functional. We present a numerical algorithm based on our earlier result that the optimal microstructure is attained by sequential laminates of rank at most two almost everywhere.

Contrary to the optimality criteria method which is, in spite of missing a firm convergence theory, commonly used in many optimal control problems, that result enables us to effectively use the classical gradient methods.

MONDAY, 10:55–11:15, SECTION A

The Darboux–Crum Transformation and Sturm–Liouville Problems with Eigenparameter Dependent Boundary Conditions

Bruce A. Watson

Abstract. The Darboux–Crum transformation will be discussed in terms of operator factorizations. The transformation forms an (almost) isospectral link between classes of Sturm–Liouville boundary value problems with boundary conditions rationally dependent on the eigenparameter. This has applications in inverse spectral problems where the boundary conditions are rationally dependent on the eigenparameter.

LIST OF PARTICIPANTS

Ibrahim Aganović, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: aga@math.hr

Nenad Antonić, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: nenad@lnr.math.hr

Ágnes Baran, Institute of Mathematics and Informatics, University of Debrecen, Egyetem sq. 1, H-4032 Debrecen, Hungary.
e–mail: szagnes@inf.unideb.hu
fax: +36 52 416-857

Sándor Baran, Faculty of Informatics, University of Debrecen, Egyetem sq. 1, H-4010 Debrecen, Hungary.
e–mail: barans@inf.unideb.hu
http://www.inf.unideb.hu/~barans
fax: +36 52 416-857

Nataša Bilić, Faculty of Civil Engineering and Architecture, University of Split, Matice hrvatske 15, 21000 Split, Croatia.
e–mail: Bozo.Vrdoljak@gradst.hr

Olivier Bodart, Laboratoire de Mathématiques CNRS (UMR 6620) Université Blaise Pascal (Clermont–Ferrand 2) 63177 Aubière cedex, France
e–mail: obodart@math.univ-bpclermont.fr

Nela Bosner, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: nela@math.hr
http://www.math.hr/~nela

Tina Bosner, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: tinab@math.hr

Krešimir Burazin, Department of Mathematics, University of Osijek, Gajev trg 6, 31000 Osijek, Croatia.
e–mail: kburazin@mathos.hr

Alain Bourgeat, UCB Lyon, UMR 5208 – MCS, Bât. ISTIL Domaine de la Doua; 15 Bd. Latarjet 69622 Villeurbanne Cedex, France.
e–mail: bourgeat@mcs.univ-lyon1.fr
http://momas.univ-lyon1.fr

Vanja Cecić, CV Sistemi d.o.o., Padovčeva 3, 10000 Zagreb, Croatia.
e–mail: vanja@cvs.hr
http://www.cvs.hr

Thierry Clopeau, Institut Camille Jordan, UFR Mathématiques, Site de Gerland Bât. A, Université Claude Bernard Lyon 1, 50, avenue Tony Garnier, 69367 Lyon Cedex 07, France.
e–mail: `Thierry.Clopeau@univ-lyon1.fr`
fax: +0033 (0)4 37 28 74 15

Bojan Crnković, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia.
e–mail: `bojan.crnkovic@riteh.hr`

Sonja Currie, ?
e–mail: `scurrie@ananzi.co.za`

Sunčica Čanić, Department of Mathematics, 622 PGH Building, University of Houston, Houston, Texas 77204-3476, USA.
e–mail: `canic@math.uh.edu`

Nelida Črnjarić–Žic, Faculty of Engineering, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia.
e–mail: `nelida@riteh.hr`

Vincent Devigne, Centre SITE, Environmental Sciences, Ecole Nationale Supérieure des Mines de Saint–Etienne, 158, cours Fauriel, 42023 Saint–Etienne Cedex 2, France.
e–mail: `Vincent.Devigne@emse.fr`
`http://www.emse.fr/~devigne`
fax: +0033 (0)4 77 42 66 33

Zlatko Drmač, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: `drmac@math.hr`
`http://www.math.hr/~drmac`

Tomislav Fratrović, Faculty of Traffic Engineering and Transport Studies, University of Zagreb, Vukelićeva 4, HR-10002 Zagreb, Croatia.
e–mail: `tomislav.fratrovic@fpz.hr`

Giovanni P. Galdi, University of Pittsburgh, 630 Benedum Engineering Hall, Pittsburgh, USA.
e–mail: `galdi@engr.pitt.edu`
`http://www.engr.pitt.edu/`

József Gáll, Faculty of Informatics, University of Debrecen, 4010 Debrecen, Pf. 12., Hungary.
e–mail: `jgall@inf.unideb.hu`

Ioan Golet, "Politehnica" Univ. of Timişoara Dept. of Mathematics, Str. Regina Maria No. 1, 300004 Romania.
e–mail: `igolet@etv.utt.ro`

Luka Grubišić, LG Mathematische Physik, Fernuniversität Hagen, Postfach 940, 58084 Hagen, Germany.
e–mail: `luka.grubisic@fernuni-hagen.de`
`http://www.fernuni-hagen.de/mathphys/`
fax: +49 2331 9874601

Hervé Guillard, INRIA, BP. 93, 06902 Sophia Antipolis Cedex, France.
e–mail: `Herve.Guillard@sophia.inria.fr`

Vjeran Hari, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: `hari@math.hr`
`http://www.math.hr/~hari`

Anders Holmbom, Mid Sweden University, Campus Östersund, Östersund, Sweden.
e–mail: `anders.holmbom@miun.se`

Dragan Jukić, Department of Mathematics, University of Osijek, Gajev trg 6, 31000 Osijek, Croatia.
e–mail: `jukicd@mathos.hr`

Mladen Jurak, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: `jurak@math.hr`
`http://http://www.math.hr/ jurak/`

Maja Karaga, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: `karaga@math.hr`
`http://www.math.hr/~karaga`

Zdenka Kolar–Begović, Department of Mathematics, University of Osijek, Gajev trg 6, 31000 Osijek, Croatia.
e–mail: `zkolard@mathos.hr`

Vedran Krčadinac, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: `krcko@math.hr`
`http://www.math.hr/~krcko`

Daniel Kressner, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: `kressner@math.hr`
`http://http://www.math.hr/ kressner/`

Saša Krešić–Jurić, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, University of Split, Rudjera Boskovića b.b., 21000 Split, Croatia.
e–mail: skresic@fesb.hr

Martin Lazar, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: martin@math.hr

Senka Maćešić, Faculty of Engineering, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia.
e–mail: senka.vukovic@ri.t-com.hr

Darija Marković, Department of Mathematics, University of Osijek, Gajev trg 6, 31000 Osijek, Croatia.
e–mail: darija@mathos.hr

Miljenko Marušić, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.
e–mail: miljenko.marusic@math.hr

Sanja Marušić, Faculty of Traffic Engineering and Transport Studies, University of Zagreb, Vukelićeva 4, 10000 Zagreb, Croatia.
e–mail: marusics@fpz.fpz.hr

Eduard Marušić–Paloka, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.
e–mail: emarusic@math.hr

Andro Mikelić, Institut Camille Jordan, UFR Mathématiques, Site de Gerland Bât. A, Université Claude Bernard Lyon 1, 50, avenue Tony Garnier, 69367 Lyon Cedex 07, France.
e–mail: Andro.Mikelic@univ-lyon1.fr
http://amikelic.free.fr
fax: +0033 (0)4 37 28 74 15

Nermina Mujaković, Department of Mathematics, Faculty of Philosophy, University of Rijeka, Omladinska 14, 51000 Rijeka, Croatia.
e–mail: mujakovic@inet.hr

Boris Muha, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: borism@math.hr

Gunter Neumann, Department of Bioinformatics, Am Abbé Platz 2, 07745 Jena, Germany.
e–mail: gfneuman@minet.uni-jena.de

Alma Omerspahić, Faculty of Mechanical Engineering, University of Sarajevo, Vilsonovo šetalište 9, 71000 Sarajevo, Bosnia and Herzegovina.
e–mail: Alma.Omerspahic@mef.unsa.ba

MariaRosaria Padula, Department of Mathematics, via Machiavelli 35, 44100 Ferrara, Italy.
e–mail: pad@unife.it

Gyula Pap, Faculty of Informatics, University of Debrecen, 4010 Debrecen, Pf. 12., Hungary.
e–mail: papgy@inf.unideb.hu
http://www.inf.unideb.hu/valseg/dolgozok/papgy/papgy.html

Mervan Pašić, Faculty of Electrical Engineering and Computing, University of Zagreb, Unska 3, 10000 Zagreb, Croatia.
e–mail: mervan.pasic@fer.hr

Igor Pažanin, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: pazanin@math.hr

Andrey L. Piatnitski, HiN, Narvik University College, P.O. Box 385, N-8505 Narvik, Norway and Lebedev Physical Institute RAS, Leninski prospect, 53. Moscow, 117333, Russia.
e–mail: andrey@sci.lebedev.ru

Olivier Pironneau, LJLL, University of Paris VI, Paris, France.
e–mail: pironneau@ann.jussieu.fr

Eugene B. Postnikov, Theoretical Physics Department, Kursk State University, Radishcheva st., 33, 305000, Kursk, Russia.
e–mail: postnicov@mail.ru
http://chaos.phys.msu.ru/postnicov/

Andrei V. Prasolov, Department of Mathematics and Statistics, University of Tromsø, N-9037 Tromsø, Norway.
e–mail: andreip@math.uit.no
http://www.math.uit.no/users/andreip

Andrijana Radovčić, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: aradov@math.hr

Andrija Raguž, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: andrija@math.hr
http://www.math.hr/~andrija

Stefan Ratschan, Max–Planck–Institut für Informatik, Stuhlsatzenhausweg 85, 66123 Saarbrücken, Germany.
e–mail: stefan.ratschan@mpi-sb.mpg.de
http://www.mpi-sb.mpg.de/∼ ratschan

Mladen Rogina, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: rogina@math.hr
http://www.math.hr/~rogina

Kristian Sabo, Department of Mathematics, University of Osijek, Gajev trg 6, 31000 Osijek, Croatia.
e–mail: ksabo@mathos.hr

Rudolf Scitovski, Department of Mathematics, University of Osijek, Gajev trg 6, 31000 Osijek, Croatia.
e–mail: scitowsk@mathos.hr

Sanja Singer, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, I. Lučića 5, 10000 Zagreb, Croatia.
e–mail: ssinger@math.hr

Saša Singer, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: singer@math.hr
http://www.math.hr/~singer

Luka Sopta, Faculty of Engineering, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia.
e–mail: luka.sopta@riteh.hr

Maja Starčević, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: mstarcev@math.hr

Nils Svanstedt, Mathematical Sciences, Chalmers University, Eklandagatan 86, Gothenburg, Sweden.
e–mail: nilss@math.chalmers.se

Josip Tambača, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: tambaca@math.hr
http://www.math.hr/~tambaca

Claudia Timofte, Department of Mathematics, Faculty of Physics, University of Bucharest, P.O. Box MG-11, Bucharest-Magurele, Romania.
e–mail: claudiatimofte@hotmail.com

Zvonimir Tutek, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: tutek@math.hr

Zlatko Udovičić, Prirodno matematički fakultet, Odsjek za matematiku, Zmaja od Bosne 35, 71000 Sarajevo, Bosnia i Herzegovina.
e–mail: zzlatko@pmf.unsa.ba

Igor Velčić, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: ivelcic@math.hr

Krešimir Veselić, LG Mathematische Physik, Fernuniversität Hagen, P.O. Box 940, 58084 Hagen, Germany.
e–mail: kresimir.veselic@fernuni-hagen.de
http://www.fernuni-hagen.de/mathphys/veselic/

Vladimir Volenec, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: volenec@math.hr

Božo Vrdoljak, Faculty of Civil Engineering, University of Split, Matice hrvatske 15, 21000 Split, Croatia.
e–mail: Bozo.Vrdoljak@gradst.hr

Marko Vrdoljak, Department of Mathematics, University of Zagreb, P.O. Box 335, 10002 Zagreb, Croatia.
e–mail: marko@math.hr

Bruce A. Watson, School of Mathematics, University of the Witwatersrand, Johannesburg, South Africa.
e–mail: bwatson@maths.wits.ac.za

Vida Zadelj–Martić, Faculty of Geodesy, University of Zagreb, Kačićeva 26, 10000 Zagreb, Croatia.
e–mail: vzadelj@geodet.geof.hr

Emil Žagar, Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia.
e–mail: Emil.Zagar@fmf.uni-lj.si